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An EPQ model for Deteriorating Items with mixed Demand pattern and Time Dependent Holding cost.

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Abstract:-The Economic Production Quantity (EPQ) model is commonly used by practitioners in the fields of production and inventory management to assist them in making decisions on production lot size. The classical EPQ model assumes that all the items produced are of perfect quality. Deterioration is the common phenomenon in the real production system of products like pharmaceuticals, food, flowers, vegetables etc. Here attempt has been made to discuss the different demand pattern for different time period. During inventory buildup time, demand is inventory dependent and during inventory depletion period, demand use is constant. Holding cost used here is linear function of time. The differential calculus method is used to find the optimal solution of the model. Convexity of the total cost with production up time has been proved. The influence of inventory dependent consumption factor is also discussed. Finally, numerical examples are used to illustrate the proposed model. Some managerial insights are also inferred from the sensitive analysis of model parameters.

Index Terms:- Deteriorating items; EPQ; Inventory dependent consumption rate; mix demand.

1.INTRODUCTION

Deterioration leads to damage of items such that they cannot be used for its original purposes. The effect of deterioration is very important in many inventory systems. Food item like, milk, pack food, meat, flowers and bakery products are items in which sufficient deterioration can take place during the normal storage period of the units. For longer storage periods require additional specialized equipment and facilities, resulting in higher holding costs. This may lead to loss in system. It would be more reasonable and realistic if it is assumed that the holding cost as a time dependent function. Hezari[1] developed EPQ model by considering imperfect and defective items. Holding cost for defective items has not been considered. Davidet. al [2] considered partial backordering with constant demand to study inventory model. Jiangtao[3] consider a multi-item inventory system, where the vendor provides the retailer with delay in payment. Yao [4] established a model for deteriorating items under delay in payment, in which the demand is a negative exponential function of price. Jia [5] reported a supply chain system with trade credit. Liao [6] established an EPQ model for deteriorating items with delay in payment. Shinn [7] developed a model with trade credit, and the length of credit period is a function of order size. Chung [8] studied a inventory system which purchaser acquired allowable delay in payment if order size is greater than a predetermined quantity. Liao [9] developed a model with delay in payment, in which there are two warehouses, one is own, another is rented. Tsao [10] considered an inventory model in supply-chain system for multiitem under the policy of trade credit. Mim,et.al [11] developed an inventory model for deteriorating items under stock-dependent demand and trade credit. Alfares [12] considered a multi-period model with stock dependent
rate under the assumption of holding cost is a function
of storage time. Yang [13] considered a model under inflation for deteriorating items with stock-dependent rate. Panda [14] developed a two warehouses model with single vendor multiple retailers when demand depends on stock and selling price.

Most of the research developed EPQ models by considering various demand patterns like stock dependent demand, power form demand, ramp type of demand, time dependent demand, selling price dependent demand and exponential demand. It is well known that, the demand rate vary with change in inventory level. From the literature review, it is observed that the different demand pattern has not been discussed by any researcher so far on different time period. The present study may be significant in filling this gap since it aims to develop EPQ model by assuming demand as inventory dependent criteria during inventory buildup time and constant during inventory depletion period. Also holding cost is considered as linear function of time. This paper has several sections. Research motivation and literature is narrated in introduction. Next section contains notations and assumption. The following sections formulate the model and

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derives the optimal solutions. The aspect of numerical and sensitivity analysis is discussed in some details followed by some concluding remarks

2. ASSUMPTIONS AND NOTATIONS

Following assumption and notations are used to develop the model.

2.1 Assumptions:-

The following assumptions are made in development of the model.

a) The production rate is known and is constant.
b) The production rate is greater than the demand.
c) The demand is inventory level dependent in up time and constant in down time.
d) Deterioration of the items is at constant rate.
e) Inventory holding cost is linear function of time.
f) Deterioration of the items start as it enters into inventory.
g) Shortages are not allowed.
h) Holding cost is linear function of time.

2.2 Notation:-

I1 – Inventory level during production up time.
I2 – Inventory level during production down time.
T1– Production up time.
T2 – Production down time.
P – Rate of production.
D – Basic demand.
θ – Rate of Deterioration
α – Inventory dependent consumption rate parameter.
H – Holding cost per unit time, H(t)=a +bt.
T – Production cycle time.
TC – Total cost.
TCT – Total cost per unit time.

3. MODEL FORMULATION

This work develop an EPQ model for deteriorating items that considers the holding cost as linear function of time. Inventories are buildup gradually during production up time. To keep the production process to its initial working condition, setup is essential. At the beginning it is assumed that the inventory is zero. During the time period (0,T1), demand rate is inventory dependent and as the production rate is greater than the demand rate, inventory is gradually buildup at a rate of (P-D). This rate is offset by a constant deterioration rate. At time T1 the inventory will be maximum. At this stage production is terminated and on hand inventory will be used to meet the demand and to offset the loss due to deterioration. During time (0,T2) demand remains constant. Inventory during the production system is shown by fig. 1.

During the time span [0, T1], Inventory level with respect to time for constant deterioration rate and inventory dependent demand, be governed by

$$\frac{dI_1(t)}{dt} = P - D - \alpha I_1(t) - \theta I_1(t) \qquad 0 \le t \le T1 \quad (1)$$

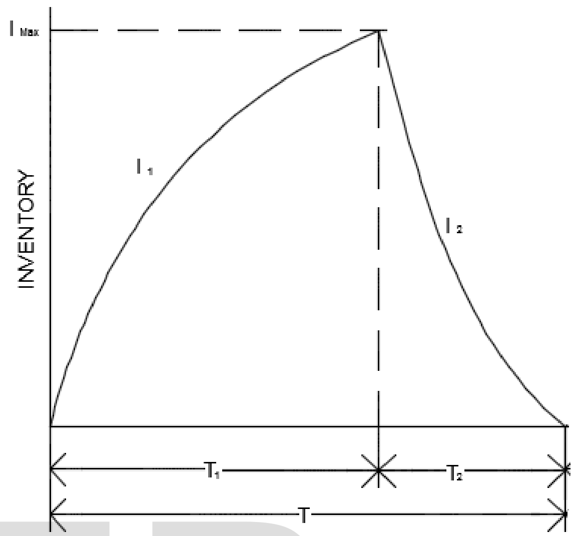

Figure 1. Inventory System

In the time interval (0,T2), the change in inventory level for combined effect of constant demand and deterioration.is governed by the following differential equation

$$\frac{dI_2(t)}{dt} = -D - \theta I_2(t) \qquad 0 \le t \le T2$$

(2)

The solution to above equations for initial boundary conditions associated with it, at t = 0, I1(t) = 0 and at t = T2 I2(t) = 0 , is as follows. These two equations are used in the derivation of our model.

$$I_1(t) = \frac{P - D}{\alpha + \theta}\left[1 - e^{-(\alpha+\theta)t}\right] \qquad 0 \le t \le T1 \quad (3)$$

$$I_2(t) = \frac{D}{\theta}\left[e^{\theta(T_2 - t)} - 1\right] \qquad 0 \le t \le T2 \quad (4)$$

From equation 3 and 4 ,by using boundary condition I1(T1) = I2(0)

$$\frac{P - D}{\alpha + \theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] = \frac{D}{\theta}\left[e^{\theta(T_2)} - 1\right]$$

$$T_2 \approx \frac{P - D}{D}\left[T_1 - \frac{(\alpha+\theta)T_1^2}{2}\right] \quad (5)$$

Total Inventory holding cost is given by

$$IH = \int^{T_1} H(t) I_1(t) dt + \int^{T2} H(t) I_2(t) dt \quad 6.$$

$$= \int^{T_1} (a + bt) I_1(t) dt + \int^{T2} (a + bt) I_2(t) dt \quad 7.$$

From equation 3 and 4.and after some simplification,

$$IH = (P-D)T_1^2\left[\frac{a}{2} + \frac{b}{\alpha+\theta}\right] + DT_2^2\left[\frac{a}{2} + \frac{b}{\theta}\right] \quad 8.$$

Total cost = Set up cost + Holding cost.
TC= A + IH.

$$TC = A + (P-D)T_1^2\left[\frac{a}{2} + \frac{b}{\alpha+\theta}\right] + DT_2^2\left[\frac{a}{2} + \frac{b}{\theta}\right] \quad 9.$$

$$= A + (P-D)T_1^2\left[\frac{a}{2} + \frac{b}{\alpha+\theta}\right] + \frac{(P-D)^2 T_1^2}{D}\left[\frac{a}{2} + \frac{b}{\theta}\right] \quad 10.$$

Production cycle time = T= T1+T2

$$TCT = \frac{TC}{T}$$

$$TCT = \frac{A + (P-D)T_1^2\left[\frac{a}{2} + \frac{b}{\alpha+\theta}\right] + \frac{(P-D)^2 T_1^2}{D}\left[\frac{a}{2} + \frac{b}{\theta}\right]}{T_1 + T_2} \quad 11$$

$$= \frac{A + T_1^2(K_1 + K_2)}{T_1 K_3}$$

The optimum production up time can be derived by satisfying the equation (12)

$$\frac{dTCT}{DT_1} = 0 \quad 12.$$

$$T_1 = \sqrt{\frac{AK_3}{K_3[K_1 + K_2]}} \quad 13.$$

Where,

$$K_1 = (P-D)\frac{a}{2}\left(1 + \frac{P-D}{D}\right)$$

$$K_2 = (P-D)b\left[\frac{1}{\alpha+\theta} + \frac{P-D}{D\theta}\right]$$

$$K_3 = \left(1 + \frac{P-D}{D}\right)$$

4.Numerical Example And Sensitivity Analysis

Numerical example and sensitivity analysis has been carried out to validate the theoretical aspects. The numerical data is adopted from Min et al.(16) except some additional data like deterioration cost, set up cost and inspection cost. Let, A= Rs.30 per production cycle, P = 2500 units per unit time, D = 1200 units per unit time, α = 0.5, θ = 0.1 ,a= 2 b = 1.5.

The optimum value of T1 can be found, as the total cost function is convex (Fig. 2). The optimum value of T1 is 0.033. The optimum total cost per unit time is TCT = Rs.866.75. Sensitivity analysis is carried out by changing each parameter by –40% to +40%, taking one parameter at a time and keeping others unchanged.

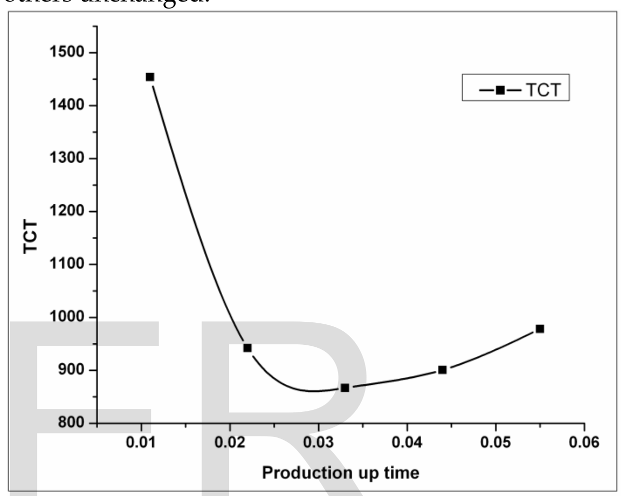

Figure 2. T_1 V/S TCT

Table 1.Sensitivity analysis of T_1.

Parameter changes				
Parameters	**-40%**	**-20%**	**20%**	**40%**
	T_1	**T_1**	**T_1**	**T_1**
P	0.115	0.051	0.024	0.019
D	0.019	0.025	0.043	0.056
α	0.032	0.032	0.033	0.033
Θ	0.026	0.03	0.035	0.037
A	0.033	0.033	0.032	0.032
B	0.041	0.036	0.03	0.028

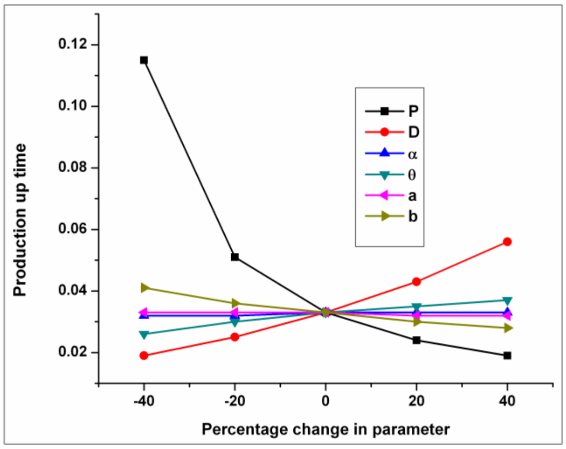

Figure 3.Parameter change V/S T_1

Table 2. Sensitivity Analysis of TCT

Parameter changes				
Parameters	-40%	-20%	20%	40%
	TCT	TCT	TCT	TCT
P	415.69	702.55	975.2	1055.68
D	874.66	893.42	805.8	712.86
α	892.38	877.38	859.38	853.85
θ	1072.21	949.18	806.8	760.6
a	849.56	858.15	876.34	884.93
b	693.44	785.01	941.7	1010.76

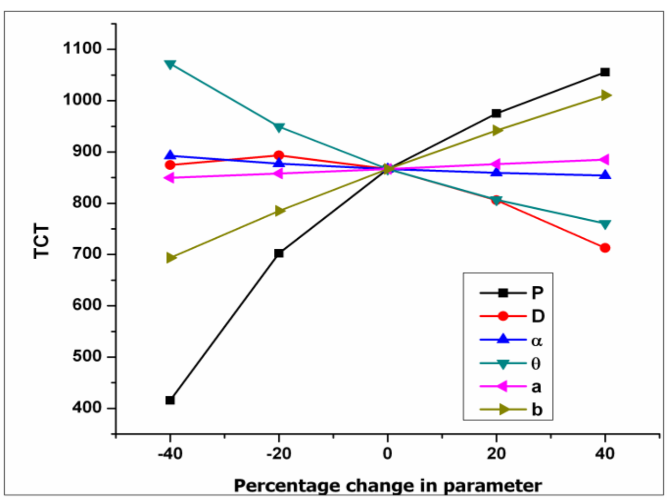

Figure 4. Parameter change V/S TCT

From fig. 3 It has been observed that production up time is highly sensitive to change in production rate and demand. Production up time is moderately sensitive to inventory demand parameter and holding cost parameter 'b'. While production time is slightly sensitive to deterioration rate and other holding cost parameter i.e 'a'. T1 increases due to increase in demand and decreases due to increase in production rate.

From fig.4 It is observed that, total cost per unit time is highly sensitive to production rate, deterioration rate and holding cost parameter 'b'. From -40% to -20% change in production rate, TCT increase in higher rate than the 20% to 40% change in production rate. This indicates that at higher production rate inventory will build up at higher rate. And as demand is inventory dependent, demand rate increases and hence TCT increases at lower rate at higher production rate. Holding cost parameter 'b' increases the TCT increases. This indicates that perishable items required special storage arrangement which increases the cost.

5.Conclusion

Here EPQ model has been developed by considering different demand pattern in different time period and holding cost as function of time. Effect of inventory dependent demand parameter has been discussed. As deterioration is common phenomenon in production system and more in perishable items, which requires special storage arrangement leads to increase in production. The present study can be useful for the inventory managers in decision making. Further model can be extended by considering different demand pattern, deterioration rate and holding cost.

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